NAG Toolbox for MATLAB

g13ca

1 Purpose

g13ca calculates the smoothed sample spectrum of a univariate time series using one of four lag windows – rectangular, Bartlett, Tukey or Parzen window.

2 Syntax

```
[c, xg, ng, stats, ifail] = g13ca(nx, mtx, px, iw, mw, ic, c, kc, l, lg,
xg, 'nc', nc, 'nxg', nxg)
```

3 Description

The smoothed sample spectrum is defined as

$$\hat{f}(\omega) = \frac{1}{2\pi}\left(C_0 + 2\sum_{k=1}^{M-1} w_k C_k \cos(\omega k)\right),$$

where M is the window width, and is calculated for frequency values

$$\omega_i = \frac{2\pi i}{L}, \qquad i = 0, 1, \ldots, [L/2],$$

where [] denotes the integer part.

The autocovariances C_k may be supplied by you, or constructed from a time series $x_1, x_2, \ldots, x_n$, as

$$C_k = \frac{1}{n}\sum_{t=1}^{n-k} x_t x_{t+k},$$

the fast Fourier transform (FFT) being used to carry out the convolution in this formula.

The time series may be mean or trend corrected (by classical least squares), and tapered before calculation of the covariances, the tapering factors being those of the split cosine bell:

$$\begin{array}{ll} \frac{1}{2}\left(1 - \cos\left(\pi\left(t - \frac{1}{2}\right)/T\right)\right), & 1 \leq t \leq T \\ \frac{1}{2}\left(1 - \cos\left(\pi\left(n - t + \frac{1}{2}\right)/T\right)\right), & n + 1 - T \leq t \leq n \\ 1, & \text{otherwise,} \end{array}$$

where $T = \left[\frac{np}{2}\right]$ and p is the tapering proportion.

The smoothing window is defined by

$$w_k = W\left(\frac{k}{M}\right), \qquad k \leq M - 1,$$

which for the various windows is defined over $0 \leq \alpha < 1$ by

rectangular:

$$W(\alpha) = 1$$

Bartlett:

$$W(\alpha) = 1 - \alpha$$

Tukey:

$$W(\alpha) = \tfrac{1}{2}(1 + \cos(\pi\alpha))$$

Parzen:

$$W(\alpha) = 1 - 6\alpha^2 + 6\alpha^3, \quad 0 \leq \alpha \leq \tfrac{1}{2}$$

$$W(\alpha) = 2(1 - \alpha)^3, \qquad \tfrac{1}{2} < \alpha < 1.$$

The sampling distribution of $\hat{f}(\omega)$ is approximately that of a scaled χ_d^2 variate, whose degrees of freedom d is provided by the function, together with multiplying limits mu, ml from which approximate 95% confidence intervals for the true spectrum $f(\omega)$ may be constructed as $\left[ml \times \hat{f}(\omega), mu \times \hat{f}(\omega)\right]$. Alternatively, $\log \hat{f}(\omega)$ may be returned, with additive limits.

The bandwidth b of the corresponding smoothing window in the frequency domain is also provided. Spectrum estimates separated by (angular) frequencies much greater than b may be assumed to be independent.

4 References

Bloomfield P 1976 *Fourier Analysis of Time Series: An Introduction* Wiley

Jenkins G M and Watts D G 1968 *Spectral Analysis and its Applications* Holden–Day

5 Parameters

5.1 Compulsory Input Parameters

1: **nx – int32 scalar**

n, the length of the time series.

Constraint: **nx** ≥ 1.

2: **mtx – int32 scalar**

If covariances are to be calculated by the function (**ic** $= 0$), **mtx** must specify whether the data are to be initially mean or trend corrected.

mtx $= 0$

For no correction.

mtx $= 1$

For mean correction.

mtx $= 2$

For trend correction.

Constraint: if **ic** $= 0$, $0 \leq$ **mtx** ≤ 2.

If covariances are supplied (**ic** $\neq 0$), **mtx** is not used.

3: **px – double scalar**

If covariances are to be calculated by the function (**ic** $= 0$), **px** must specify the proportion of the data (totalled over both ends) to be initially tapered by the split cosine bell taper.

If covariances are supplied (**ic** $\neq 0$), **px** must specify the proportion of data tapered before the supplied covariances were calculated and after any mean or trend correction. **px** is required for the calculation of output statistics. A value of 0.0 implies no tapering.

Constraint: $0.0 \leq$ **px** ≤ 1.0.

4: **iw – int32 scalar**

The choice of lag window. **iw** $= 1$ for rectangular, 2 for Bartlett, 3 for Tukey or 4 for Parzen.

Constraint: $1 \leq$ **iw** ≤ 4.

5: **mw – int32 scalar**

M, the 'cut-off' point of the lag window. Windowed covariances at lag M or greater are zero.

Constraint: $1 \leq$ **mw** $\leq$ **nx**.

6: **ic – int32 scalar**

Indicates whether covariances are to be calculated in the function or supplied in the call to the function.

ic $= 0$

Covariances are to be calculated.

ic $\neq 0$

Covariances are to be supplied.

7: **c(nc) – double array**

If **ic** $\neq 0$, **c** must contain the **nc** covariances for lags from 0 to (**nc** $- 1$), otherwise **c** need not be set.

8: **kc – int32 scalar**

If **ic** $= 0$, **kc** must specify the order of the fast Fourier transform (FFT) used to calculate the covariances. **kc** should be a product of small primes such as 2^m where m is the smallest integer such that $2^m \geq$ **nx** $+$ **nc**, provided $m \leq 20$.

If **ic** $\neq 0$, that is covariances are supplied, **kc** is not used.

Constraint: **kc** $\geq$ **nx** $+$ **nc**. The largest prime factor of **kc** must not exceed 19, and the total number of prime factors of **kc**, counting repetitions, must not exceed 20. These two restrictions are imposed by c06ea which performs the FFT

9: **l – int32 scalar**

L, the frequency division of the spectral estimates as $\frac{2\pi}{L}$. Therefore it is also the order of the FFT used to construct the sample spectrum from the covariances. **l** should be a product of small primes such as 2^m where m is the smallest integer such that $2^m \geq 2M - 1$, provided $m \leq 20$.

Constraint: **l** $\geq 2 \times$ **mw** $- 1$. The largest prime factor of **l** must not exceed 19, and the total number of prime factors of **l**, counting repetitions, must not exceed 20. These two restrictions are imposed by c06ea which performs the FFT

10: **lg – int32 scalar**

Indicates whether unlogged or logged spectral estimates and confidence limits are required.

lg $= 0$

Unlogged.

lg $= 1$

Logged.

Constraint: **lg** $= 0$ or 1.

11: **xg(nxg) – double array**

If the covariances are to be calculated, then **xg** must contain the **nx** data points. If covariances are supplied, **xg** may contain any values.

5.2 Optional Input Parameters

1: **nc – int32 scalar**

Default: The dimension of the array **c**.

the number of covariances to be calculated in the function or supplied in the call to the function.

Constraint: **mw** $\leq$ **nc** $\leq$ **nx**.

2: **nxg – int32 scalar**

Default: The dimension of the array **xg**.

Constraints:

if **ic** $= 0$, **nxg** $\geq \max($**kc**, **l**$)$;
if **ic** $\neq 0$, **nxg** $\geq$ **l**.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **c(nc) – double array**

If **ic** $= 0$, **c** will contain the **nc** calculated covariances.

If **ic** $\neq 0$, the contents of **c** will be unchanged.

2: **xg(nxg) – double array**

Contains the **ng** spectral estimates, $\hat{f}(\omega_i)$, for $i = 0, 1, \ldots, [L/2]$ in **xg**(1) to **xg**(**ng**) respectively (logged if **lg** $= 1$). The elements **xg**(i), for $i =$ **ng** $+ 1, \ldots,$ **nxg** contain 0.0.

3: **ng – int32 scalar**

The number of spectral estimates, $[L/2] + 1$, in **xg**.

4: **stats(4) – double array**

Four associated statistics. These are the degrees of freedom in **stats**(1), the lower and upper 95% confidence limit factors in **stats**(2) and **stats**(3) respectively (logged if **lg** $= 1$), and the bandwidth in **stats**(4).

5: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, **nx** < 1,
or **mtx** < 0 and **ic** $= 0$,
or **mtx** > 2 and **ic** $= 0$,
or **px** < 0.0,
or **px** > 1.0,
or **iw** < 1,
or **iw** > 4,
or **mw** < 1,
or **mw** $>$ **nx**,

or **nc** $<$ **mw**,
or **nc** $>$ **nx**,
or **nxg** $<$ max(**kc**, **l**) and **ic** $= 0$,
or **nxg** $<$ **l** and **ic** $\neq 0$.

ifail $= 2$

On entry, **kc** $<$ **nx** $+$ **nc**,
or **kc** has a prime factor exceeding 19,
or **kc** has more than 20 prime factors, counting repetitions.

This error only occurs when **ic** $= 0$.

ifail $= 3$

On entry, **l** $< 2 \times$ **mw** $- 1$,
or **l** has a prime factor exceeding 19,
or **l** has more than 20 prime factors, counting repetitions.

ifail $= 4$

One or more spectral estimates are negative. Unlogged spectral estimates are returned in **xg**, and the degrees of freedom, unlogged confidence limit factors and bandwidth in **stats**.

ifail $= 5$

The calculation of confidence limit factors has failed. This error will not normally occur. Spectral estimates (logged if requested) are returned in **xg**, and degrees of freedom and bandwidth in **stats**.

7 Accuracy

The FFT is a numerically stable process, and any errors introduced during the computation will normally be insignificant compared with uncertainty in the data.

8 Further Comments

g13ca carries out two FFTs of length **kc** to calculate the covariances and one FFT of length **l** to calculate the sample spectrum. The time taken by the function for an FFT of length n is approximately proportional to $n \log(n)$ (see Section 8 of the document for c06ea for further details).

9 Example

```
nx = int32(256);
mtx = int32(1);
px = 0.1;
iw = int32(4);
mw = int32(100);
ic = int32(0);
c = zeros(100, 1);
kc = int32(360);
l = int32(200);
lg = int32(0);
xg = zeros(360,1);
xg(1:256) = [5;
     11;
     16;
     23;
     36;
     58;
     29;
     20;
     10;
```

```
    8;
    3;
    0;
    0;
    2;
    11;
    27;
    47;
    63;
    60;
    39;
    28;
    26;
    22;
    11;
    21;
    40;
    78;
    122;
    103;
    73;
    47;
    35;
    11;
    5;
    16;
    34;
    70;
    81;
    111;
    101;
    73;
    40;
    20;
    16;
    5;
    11;
    22;
    40;
    60;
    80.90000000000001;
    83.40000000000001;
    47.7;
    47.8;
    30.7;
    12.2;
    9.6;
    10.2;
    32.4;
    47.6;
    54;
    62.9;
    85.90000000000001;
    61.2;
    45.1;
    36.4;
    20.9;
    11.4;
    37.8;
    69.8;
    106.1;
    100.8;
    81.59999999999999;
    66.5;
    34.8;
    30.6;
    7;
    19.8;
    92.5;
    154.4;
```

```
125.9;
84.8;
68.09999999999999;
38.5;
22.8;
10.2;
24.1;
82.90000000000001;
132;
130.9;
118.1;
89.90000000000001;
66.59999999999999;
60;
46.9;
41;
21.3;
16;
6.4;
4.1;
6.8;
14.5;
34;
45;
43.1;
47.5;
42.2;
28.1;
10.1;
8.1;
2.5;
0;
1.4;
5;
12.2;
13.9;
35.4;
45.8;
41.1;
30.1;
23.9;
15.6;
6.6;
4;
1.8;
8.5;
16.6;
36.3;
49.6;
64.2;
67;
70.90000000000001;
47.8;
27.5;
8.5;
13.2;
56.9;
121.5;
138.3;
103.2;
85.7;
64.59999999999999;
36.7;
24.2;
10.7;
15;
40.1;
61.5;
98.5;
124.7;
```

```
96.3;
66.59999999999999;
64.5;
54.1;
39;
20.6;
6.7;
4.3;
22.7;
54.8;
93.8;
95.8;
77.2;
59.1;
44;
47;
30.5;
16.3;
7.3;
37.6;
74;
139;
111.2;
101.6;
66.2;
44.7;
17;
11.3;
12.4;
3.4;
6;
32.3;
54.3;
59.7;
63.7;
63.5;
52.2;
25.4;
13.1;
6.8;
6.3;
7.1;
35.6;
73;
85.09999999999999;
78;
64;
41.8;
26.2;
26.7;
12.1;
9.5;
2.7;
5;
24.4;
42;
63.5;
53.8;
62;
48.5;
43.9;
18.6;
5.7;
3.6;
1.4;
9.6;
47.4;
57.1;
103.9;
80.59999999999999;
```

```
     63.6;
     37.6;
     26.1;
     14.2;
     5.8;
     16.7;
     44.3;
     63.9;
     69;
     77.8;
     64.90000000000001;
     35.7;
     21.2;
     11.1;
     5.7;
     8.699999999999999;
     36.1;
     79.7;
     114.4;
     109.6;
     88.8;
     67.8;
     47.5;
     30.6;
     16.3;
     9.6;
     33.2;
     92.59999999999999;
     151.6;
     136.3;
     134.7;
     83.9;
     69.4;
     31.5;
     13.9;
     4.4;
     38];
[cOut, xgOut, ng, stats, ifail] = g13ca(nx, mtx, px, iw, mw, ic, c, kc,
l, lg, xg)
```

```
cOut =
     array elided
xgOut =
     array elided
ng =
         101
stats =
    9.0000
    0.4731
    3.3329
    0.1165
ifail =
           0
```